

False loss of coherence

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Published in **Relativistic Quantum Measurement and Decoherence,**

ed F. Petruccione , Springer Verlag(2000)

Abstract

The loss of coherence of a quantum system coupled to a heat bath as expressed by the reduced density matrix is shown to lead to the miss-characterization of some systems as being incoherent when they are not. The spin boson problem and the harmonic oscillator with massive scalar field heat baths are given as examples of reduced incoherent density matrices which nevertheless still represent perfectly coherent systems.

I. MASSIVE FIELD HEAT BATH AND A TWO LEVEL SYSTEM

How does an environment affect the quantum nature of a system? The standard technique is to look at the reduced density matrix, in which one has traced out the environment variables. If this changes from a pure state to a mixed state (entropy $Tr\rho\ln\rho$ not equal to zero) one argues that the system has lost quantum coherence, and quantum interference effects are suppressed . However this criterion is too strong. There are couplings to the environment which are such that this reduced density matrix has a high entropy, while the system alone retains virtually all of its original quantum coherence certain experiments.

The key idea is that the external environment can be different for different states of the system. There is a strong correlation between the system and the environment. As usual, such correlations lead to decoherence in the reduced density matrix. However, the environment in these cases is actually tied to the system, and is adiabatically dragged along by the system. Thus although the state of the environment is different for the two states, one can manipulate the system alone so as to cause these apparently incoherent states to interfere with each other. One simply causes a sufficiently slow change in the system so as to drag the environment variables into common states so the quantum interference of the system can again manifest itself.

An example is if one looks at an electron with its attached electromagnetic field. Consider the electron at two different positions. The static coulomb field of the two charges differ, and thus the states of the electromagnetic field differ with the electron in the two positions. These differences can be sufficient to cause the reduced electron wave function loose coherence for a state which is a coherent sum of states located at these two positions. However, if one causes the system to evolve so as to cause the electron in those two positions to come together (eg, by having a force field such that the electron in both positions to be brought together at some central point for example), those two apparently incoherent states will interfere, demonstrating that the loss of coherence was not real.

Another example is light propagating through a slab of glass. If one simply looks at the electromagnetic field, and traces out over the states of the atoms in the glass, the light beams traveling through two separate regions of the glass will clearly decohere– the reduced density matrix for the electromagnetic field will lose coherence in position space– but those two beams of light will also clearly interfere when they exit the glass or even when they are

within the glass.

The above is not to be taken as proof, but as a motivation for the further investigation of the problem. The primary example I will take will be of a spin $\frac{1}{2}$ particle (or other two level system). I will also examine a harmonic oscillator as the system of interest. In both cases, the heat bath will be a massive one dimensional scalar field. This heat bath is of the general Caldeira Leggett type [1] (and in fact is entirely equivalent to that model in general). The mass of the scalar field will be taken to be larger than the inverse time scale of the dynamical behaviour of the system. This is not to be taken as an attempt to model some real heat bath, but to display the phenomenon in its clearest form. Realistic heat baths will in general also have low frequency excitations which will introduce other phenomena like damping and genuine loss of coherence into the problem.

II. SPIN- $\frac{1}{2}$ SYSTEM

Let us take as our first example that of a spin- $\frac{1}{2}$ system coupled to an external environment. We will take this external environment to be a one dimensional massive scalar field. The coupling to the spin system will be via purely the 3 component of the spin. I will use the velocity coupling which I have used elsewhere as a simple example of an environment (which for a massless field is completely equivalent to the Caldeira Leggett model). The Lagrangian is

$$L = \int \frac{1}{2} ((\dot{\phi}(x))^2 - (\phi(x)')^2 + m^2 \phi(x)^2 + 2\epsilon \dot{\phi}(x) h(x) \sigma_3) dx \quad (1)$$

which gives the Hamiltonian

$$H = \int \frac{1}{2} ((\pi(x) - \epsilon h(x) \sigma_3)^2 + (\phi(x)')^2 + m^2 \phi(x)^2) dx \quad (2)$$

$h(x)$ is the interaction range function, and its Fourier transform is related to the spectral response function of Leggett and Caldeira.

This system is easily solvable. I will look at this system in the following way. Start initially with the field in its free ($\epsilon = 0$) vacuum state, and the system is in the $+1$ eigenstate of σ_1 . I will start with the coupling ϵ initially zero and gradually increase it to some large value. I will look at the reduced density matrix for the system, and show that it reduces one which is almost the identity matrix (the maximally incoherent density matrix) for strong coupling.

Now I let ϵ slowly drop to zero again. At the end of the procedure, the state of the system will again be found to be in the original eigenstate of σ_1 . The intermediate maximally incoherent density matrix would seem to imply that the system no longer has any quantum coherence. However this lack of coherence is illusionary. Slowly decoupling the system from the environment should in the usual course simply maintain the incoherence of the system. Yet here, as if by magic, an almost completely incoherent density matrix magically becomes coherent when the system is decoupled from the environment.

In analyzing the system, I will look at the states of the field corresponding to the two possible σ_3 eigenstates of the system. These two states of the field are almost orthogonal for strong coupling. However they correspond to fields tightly bound to the spin system. As the coupling is reduced, the two states of the field adiabatically come closer and closer together until finally they coincide when ϵ is again zero. The two states of the environment are now the same, there is no correlation between the environment and the system, and the system regains its coherence.

The density matrix for the spin system can always be written as

$$\rho(t) = \frac{1}{2}(1 + \vec{\rho}(t) \cdot \vec{\sigma}) \quad (3)$$

where

$$\vec{\rho}(t) = Tr(\vec{\sigma}\rho(t)) \quad (4)$$

We have

$$\vec{\rho}(t) = Tr\left(\vec{\sigma}\mathcal{T}[e^{-i\int_0^t H dt}]\frac{1}{2}(1 + \vec{\rho}(0) \cdot \vec{\sigma})R_0\mathcal{T}[e^{-i\int H dt}]^\dagger\right) \quad (5)$$

where R_0 is the initial density matrix for the field (assumed to be the vacuum), and $\mathcal{T}[]$ is the time ordering operator. (Because ϵ and thus H is time dependent, the H at different times do not commute. this leads to requirement for the time ordering in the expression. As usual, the time ordered integral is the way of writing the time ordered product $\prod_n e^{-iH(t_n)dt} = e^{-iH(t)dt}e^{-iH(t-dt)dt}....e^{-iH(0)dt}.$)

Let us first calculate $\rho_3(t)$. We have

$$\rho_3(t) = Tr\left(\sigma_3\mathcal{T}[e^{-i\int_0^t H dt}]\frac{1}{2}(1 + \vec{\rho}(0) \cdot \vec{\sigma})R_0\mathcal{T}[e^{-i\int H dt}]^\dagger\right) \quad (6)$$

$$= Tr\left(\mathcal{T}[e^{-i\int_0^t H dt}]\sigma_3\frac{1}{2}(1 + \vec{\rho}(0) \cdot \vec{\sigma})R_0\mathcal{T}[e^{-i\int H dt}]^\dagger\right) \quad (7)$$

$$= Tr\left(\sigma_3\frac{1}{2}(1 + \vec{\rho}(0) \cdot \vec{\sigma})R_0\right) \quad (8)$$

$$= \rho_3(0) \quad (9)$$

because σ_3 commutes with $H(t)$ for all t . We now define

$$\sigma_+ = \frac{1}{2}(\sigma_1 + i\sigma_2) = |+\rangle\langle -|; \quad \sigma_- = \sigma_+^\dagger \quad (10)$$

Using $\sigma_+\sigma_3 = -\sigma_+$ and $\sigma_3\sigma_+ = \sigma_+$ we have

$$\begin{aligned} Tr & \left(\sigma_+ \mathcal{T}[e^{-i\int_0^t H dt}] \frac{1}{2}(1 + \vec{\rho}(0) \cdot \vec{\sigma}) R_0 \mathcal{T}[e^{-i\int H dt}]^\dagger \right) \\ = & Tr_\phi \left(\mathcal{T}[e^{-i\int (H_0 - \epsilon(t) \int \pi(x)h(x)dx) dt}]^\dagger \right. \\ & \left. \mathcal{T}[e^{-i\int (H_0 + \epsilon(t) \int \pi(x)h(x)dx) dt}] \right) < -\frac{1}{2}(1 + \vec{\rho}(0) \cdot \vec{\sigma})|+\rangle \\ = & (\rho_1(0) + i\rho_2(0))J(t) \end{aligned} \quad (11)$$

where H_0 is the Hamiltonian with $\epsilon = 0$, i.e., the free Hamiltonian for the massless scalar field and

$$J(t) = Tr_\phi \left(\mathcal{T}[e^{-i\int (H_0 - \epsilon(t) \int \pi(x)h(x)dx) dt}]^\dagger \mathcal{T}[e^{-i\int (H_0 + \epsilon(t) \int \pi(x)h(x)dx) dt}] R_0 \right) \quad (12)$$

Breaking up the time ordered product in the standard way into a large number of small time steps, using the fact that $e^{-i\epsilon(t) \int h(x)\phi(x)dx}$ is the displacement operator for the field momentum through a distance of $\epsilon(t)h(x)$, and commuting the free field Hamiltonian terms through, this can be written as

$$\begin{aligned} J(t) = & Tr_\phi \left(e^{-i\epsilon(0)\Phi(0)} \prod_{n=1}^{t/dt} \left[e^{-i(\epsilon(t_n) - \epsilon(t_{n-1}))\Phi(t_n)} \right] \right. \\ & \left. e^{i\epsilon(t)\Phi(t)} e^{i\epsilon(t)\Phi(t)} \prod_{n=t/dt}^1 \left[e^{i(\epsilon(t_n) - \epsilon(t_{n-1}))\Phi(t_n)} \right] e^{i\epsilon(0)\Phi(0)} R_0 \right) \end{aligned} \quad (13)$$

where $t_n = ndt$ and dt is a very small value, $\Phi(t) = \int h(x)\phi(t, x)dx$ and $\phi_0(t, x)$ is the free field Heisenberg field operator. Using the Campbell-Baker-Hausdorff formula, realizing that the commutators of the Φ s are c-numbers, and noticing that these c-numbers cancel between the two products, we finally get

$$J(t) = Tr_\phi \left(e^{2i(\epsilon(t)\Phi(t) - \epsilon(0)\Phi(0) + \int_0^t \dot{\epsilon}(t')\Phi(t')dt')} R_0 \right) \quad (14)$$

from which we get

$$\ln(J(t)) = -2Tr_\phi \left(R_0 \left(\epsilon(t)\Phi(t) - \epsilon(0)\Phi(0) + \int_0^t \dot{\epsilon}(t')\Phi(t')dt' \right)^2 \right) \quad (15)$$

I will assume that $\epsilon(0) = 0$, and that $\dot{\epsilon}(t)$ is very small, and that it can be neglected. (The neglected terms are of the form

$$\int \int \dot{\epsilon}^2 < \Phi(t')\Phi(t'') > dt' dt'' \approx \dot{\epsilon}^2 t \tau < \Phi(0)^2 >$$

which for a massive scalar field has τ , the coherence time scale, $\approx 1/m$. Thus, as we let $\dot{\epsilon}$ go to zero these terms go to zero.)

We finally have

$$\begin{aligned} \ln(J(t)) &= -2\epsilon(t)^2 < \Phi(t)^2 > \\ &= -2\epsilon(t)^2 \int |\hat{h}(k)|^2 \frac{1}{\sqrt{(k^2 + m^2)}} dk \end{aligned} \quad (16)$$

Choosing $\hat{h}(k) = e^{-\Gamma|k|/2}$, we finally get

$$\ln(J(t)) = -4 \int_0^\infty \epsilon(t)^2 \frac{e^{-\Gamma|k|} dk}{\sqrt{(k^2 + m^2)}} \quad (17)$$

This goes roughly as $\ln(\Gamma m)$ for small Γm , (which I will assume is true). For Γ sufficiently small, this makes J very small, and the density matrix reduces to essentially diagonal form ($\rho_z(t) \approx \rho_y(t) \approx 0$, $\rho_z(t) = \rho_z(0)$.)

However it is clear that if $\epsilon(t)$ is now lowered slowly to zero, the decoherence factor J goes back to unity, since it depends only on $\epsilon(t)$. The density matrix now has exactly its initial form again. The loss of coherence at the intermediate times was illusionary. By decoupling the system from the environment after the coherence had been lost, the coherence is restore. this is in contrast with the naive expectation in which the loss of coherence comes about because of the correlations between the system and the environment. Decoupling the system from the environment should not in itself destroy that correlation, and should not reestablish the coherence.

The above approach, while giving the correct results, is not very transparent in explaining what is happening. Let us therefor take a different approach. Let us solve the Heisenberg equations of motion for the field $\phi(t, x)$. The equations are (after eliminating π)

$$\partial_t^2 \phi(t, x) - \partial_x^2 \phi(t, x) + m^2 \phi(t, x) = -\dot{\epsilon}(t) \sigma_3 h(x) \quad (18)$$

$$\pi(t, x) = \dot{\phi}(t, x) + \epsilon(t) h(x) \sigma_3 \quad (19)$$

If ϵ is slowly varying in time, we can solve this approximately by

$$\phi(t, x) = \phi_0(t, x) + \dot{\epsilon}(t) \int \frac{1}{2m} e^{-m|x-x'|} h(x') dx' \sigma_3 + \psi(t, x) \epsilon(0) \sigma_3 \quad (20)$$

$$\pi(t, x) = \dot{\phi}_0(t, x) + \epsilon(t) h(x) \sigma_3 + \dot{\psi}(t, x) \epsilon(0) \sigma_3 \quad (21)$$

where $\phi_0(t, x)$ and $\pi_0(t, x)$ are free field solution to the equations of motion in absence of the coupling, with the same initial conditions

$$\dot{\phi}_0(0, x) = \pi(0, x) \quad (22)$$

$$\phi_0(0, x) = \phi(0, x) \quad (23)$$

, while ψ is also a solution of the free field equations but with initial conditions

$$\psi(0, x) = 0 \quad (24)$$

$$\dot{\psi}(0, x) = -h(x). \quad (25)$$

If we examine this for the two possible eigenstates of σ_3 , we find the two solutions

$$\phi_{\pm}(t, x) \approx \phi_0(t, x) \pm (\dot{\epsilon}(t) \int \frac{1}{2m} e^{-m|x-x'|} h(x') dx' + \psi(t, x)) \quad (26)$$

$$\pi_{\pm}(t, x) \approx \dot{\phi}_0(t, x) + O(\dot{\epsilon}) \pm (\epsilon(t)h(x) + \epsilon(0)\dot{\psi}(t, x)) \quad (27)$$

These solutions neglect terms of higher derivatives in ϵ . The state of the field is the vacuum state of ϕ_0, π_0 . ϕ_{\pm} and π_{\pm} are equal to this initial field plus c number fields. Thus in terms of the ϕ_{\pm} and π_{\pm} , the state is a coherent state with non-trivial displacement from the vacuum. Writing the fields in terms of their creation and annihilation operators,

$$\phi_{\pm}(t, x) = \int A_{k\pm}(t) e^{ikx} + A_{k\pm}^{\dagger} e^{-ikx} \frac{dk}{\sqrt{2\pi\omega_k}} \quad (28)$$

$$\pi_{\pm}(t, x) = i \int A_{k\pm}(t) e^{ikx} - A_{k\pm}^{\dagger} e^{-ikx} \sqrt{\frac{k^2 + m^2}{2\pi}} dk \quad (29)$$

we find that we can write $A_{k\pm}$ in terms of the initial operators A_{k0} as

$$A_{k\pm}(t) \approx A_{k0} e^{-i\omega_k t} \pm \frac{1}{2} i (\epsilon(t) - \epsilon(0) e^{-i\omega_k t}) (h(k)/\sqrt{\omega_k} + O(\dot{\epsilon}(t))) \quad (30)$$

where $\omega_k = \sqrt{k^2 + m^2}$. Again I will neglect the terms of order $\dot{\epsilon}$ in comparison with the ϵ terms. Since the state is the vacuum state with respect to the initial operators A_{k0} , it will be a coherent state with respect to the operators $A_{k\pm}$, the annihilation operators for the field at time t . We thus have two possible coherent states for the field, depending on whether the spin is in the upper or lower eigenstate of σ_3 . But these two coherent states will have a small overlap. If $A|\alpha\rangle = \alpha|\alpha\rangle$ then we have

$$|\alpha\rangle = e^{\alpha A^{\dagger} - |\alpha|^2/2} |0\rangle \quad (31)$$

Furthermore, if we have two coherent states $|\alpha\rangle$ and $|\alpha'\rangle$, then the overlap is given by

$$\langle \alpha | \alpha' \rangle = \langle 0 | e^{\alpha^* A - |\alpha|^2/2} e^{\beta A^\dagger - |\beta|^2/2} | 0 \rangle = e^{\alpha^* \beta - (|\alpha|^2 + |\beta|^2)/2} \quad (32)$$

In our case, taking the two states $|\pm_\phi\rangle$, these correspond to coherent states with

$$\alpha = -\alpha' = \frac{1}{2}i(\epsilon(t) - \epsilon(0)e^{-i\omega_k t}) = \frac{1}{2}i\epsilon(t)h(k)/\sqrt{\omega_k} \quad (33)$$

Thus we have

$$\langle +_\phi, t | -_\phi, t \rangle = \prod_k e^{-\epsilon(t)^2 |h(k)|^2 / (k^2 + m^2)} = e^{-\epsilon(t)^2 \int \frac{|h(k)|^2}{\omega_k} dk} = J(t). \quad (34)$$

Let us assume that we began with the state of the spin as $\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$. The state of the system at time t in the Schrodinger representation is $\frac{1}{\sqrt{2}}(|+\rangle + |+\phi(t)\rangle + |-\rangle + |-\phi(t)\rangle)$ and the reduced density matrix is

$$\rho = \frac{1}{2}(|+\rangle\langle +| + |-\rangle\langle -| + J^*(t)|+\rangle\langle -| + J(t)|-\rangle\langle +|). \quad (35)$$

The off diagonal terms of the density matrix are suppressed by the function $J(t)$. $J(t)$ however depends only on $\epsilon(t)$ and thus, as long as we keep ϵ small, the loss of coherence represented by J can be reversed simply by decoupling the system from the environment slowly.

The apparent decoherence comes about precisely because the system in either the two eigenstates of σ_3 drives the field into two different coherent states. For large ϵ , these two states have small overlap. However, this distortion of the state of the field is tied to the system. π changes only locally, and the changes in the field caused by the system do not radiate away. As ϵ slowly changes, this bound state of the field also slowly changes in concert. However if one examines only the system, one sees a loss of coherence because the field states have only a small overlap with each other.

The behaviour is very different if the system or the interaction changes rapidly. In that case the decoherence can become real. As an example, consider the above case in which $\epsilon(t)$ suddenly is reduced to zero. In that case, the field is left as a free field, but a free field whose state (the coherent state) depends on the state of the system. In this case the field radiates away as real (not bound) excitations of the scalar field. The correlations with the system are carried away, and even if the coupling were again turned on, the loss of coherence would be permanent.

III. OSCILLATOR

For the harmonic oscillator coupled to a heat bath, the Hamiltonian can be taken as

$$H = \frac{1}{2} \int (\pi(x) - \epsilon(t)q(t)\tilde{h}(x))^2 + (\partial_x \phi(x))^2 + m^2 \phi(t, x)^2 dx + \frac{1}{2}(p^2 + \Omega^2 q^2) \quad (36)$$

Let us assume that m is much larger than Ω or that the inverse time rate of change of ϵ . The solution for the field is given by

$$\phi(t, x) \approx \phi_0(t, x) + \psi(t, x)\epsilon(0)q(0) - \overline{\epsilon(t)q(t)} \int \frac{e^{-m|x-x'|}}{2m} h(x') dx' \quad (37)$$

$$\pi(t, x) \approx \dot{\phi}(t, x) + \dot{\psi}(t, x)\epsilon(0)q(0) - \overline{\epsilon(t)q(t)} \int \frac{e^{-m|x-x'|}}{2m} h(x') dx' + \epsilon(t)q(t)h(x) \quad (38)$$

where again ϕ_0 is the free field operator, ψ is a free field solution with $\psi(0) = 0$, $\dot{\psi}(0) = -h(x)$. Retaining terms only of the lowest order in ϵ

$$\psi(t, x) \approx \phi_0(t, x) \quad (39)$$

$$\pi(t, x) \approx \dot{\phi}(t, x) + \epsilon(t)q(t)h(x) \quad (40)$$

The equation of motion for q is

$$\dot{q}(t) = p(t) \quad (41)$$

$$\dot{p}(t) = -\Omega^2 q + \epsilon(t)\dot{\Phi}(t) \quad (42)$$

where $\Phi(t) = \int h(x)\phi(t, x)dx$. Substitution in the expression for ϕ , we get

$$\ddot{q}(t) + \Omega^2 q(t) \approx \epsilon(\dot{\Phi}_0(t)) + \epsilon(t)\overline{\epsilon(t)q(t)} \int \int h(x)h(x') \frac{e^{-m|x-x'|}}{2m} dx dx' \quad (43)$$

Neglecting the derivatives of ϵ (i.e., assuming that ϵ changes slowly even on the time scale of $1/\Omega$), this becomes

$$\left(1 + \epsilon(t)^2 \int \int h(x)h(x') \frac{e^{-m|x-x'|}}{2m} dx dx'\right) \ddot{q} + \Omega^2 q = \partial_t(\epsilon(t)\Phi(t)) \quad (44)$$

The interaction with the field thus renormalizes the mass of the oscillator to

$$M = \left(1 + \epsilon(t)^2 \int \int h(x)h(x')\right)$$

The solution for q is thus

$$q(t) \approx q(0) \cos\left(\int_0^t \tilde{\Omega}(t) dt\right) + \frac{1}{\tilde{\Omega}} \sin\left(\int_0^t \tilde{\Omega}(t) dt\right) p(0) + \frac{1}{\tilde{\Omega}} \int_0^t \sin\left(\int_{t'}^t \tilde{\Omega}(t) dt\right) \partial_t(\epsilon(t')\overline{\epsilon(t)\dot{\Phi}_0(t')}) dt' \quad (45)$$

where $\tilde{\Omega}(t) \approx \Omega/\sqrt{M(t)}$.

The important point is that the forcing term dependent on Φ_0 is a rapidly oscillating term of frequency at least m . Thus if we look for example at $\langle q^2 \rangle$, the deviation from the free evolution of the oscillator (with the renormalized mass) is of the order of $\int \sin(\tilde{\Omega}t - t') \sin(\omega(t - t'')) \langle \dot{\Phi}_0(t') \dot{\Phi}_0(t'') \rangle dt' dt''$. But $\langle \dot{\Phi}_0(t') \dot{\Phi}_0(t'') \rangle$ is a rapidly oscillating function of frequency at least m , while the rest of the integrand is a slowly varying function with frequency much less than m , Thus this integral will be very small (at least $\tilde{\Omega}/m$ but typically much smaller than this depending on the time dependence of ϵ). Thus the deviation of $q(t)$ from the free motion will in general be very very small, and I will neglect it.

Let us now look at the field. The field is put into a coherent state which depends on the value of q , because $\pi(t, x) \approx \dot{\phi}_0(t, x) + \epsilon(t)q(t)h(x)$ Thus

$$A_k(t) \approx a_{0k} e^{-i\omega_k t} + i \frac{1}{2} \hat{h}(k) \epsilon(t) q(t) / \omega_k \quad (46)$$

The overlap integral for these coherent states with various values of q is

$$\prod_k \langle i \frac{1}{2} \hat{h}(k) \epsilon(t) q / \omega_k | i \frac{1}{2} \hat{h}(k) \epsilon(t) q' / \omega_k \rangle = e^{-\frac{1}{8} \int |\hat{h}(k)|^2 dk (q - q')^2} \quad (47)$$

The density matrix for the Harmonic oscillator is thus

$$\rho(q, q') = \rho_0(t, q, q') e^{-\frac{1}{8} \int |\hat{h}(k)|^2 dk (q - q')^2} \quad (48)$$

where ρ_0 is the density matrix for a free harmonic oscillator (with the renormalized mass).

Ie, we see a strong loss of coherence of the off diagonal terms of the density matrix. However this loss of coherence is false. If we take the initial state for example with two packets widely separated in space, these two packets will loose their coherence. However, as time proceeds, the natural evolution of the Harmonic oscillator will bring those two packets together ($q - q'$ small across the wave packet). For the free evolution they would then interfere. They still do. The loss of coherence which was apparent when the two packets were widely separated disappears, and the two packets interfere just as if there were no coupling to the environment. The effect of the particular environment used is thus to renormalise the mass, and to make the density matrix appear to loose coherence.

IV. SPIN BOSON PROBLEM

Let us now complicate the spin problem in the first section by introducing into the system a free Hamiltonian for the spin as well as the coupling to the environment. Following the example of the spin boson problem, let me introduce a free Hamiltonian for the spin of the form $\frac{1}{2}\Omega\sigma_1$, whose effect is to rotate the σ_3 states (or to rotate the vector $\vec{\rho}$ in the 2 – 3 plane with frequency Ω).

The Hamiltonian now is

$$H = \frac{1}{2} \left(\int (\pi(t, x) - \epsilon(t)h(x)\sigma_3)^2 + (\partial_x\phi(x))^2 + m^2\phi(t, x)^2 dx + \Omega\sigma_1 \right) \quad (49)$$

where again $\epsilon(t)$ is a slowly varying function of time. We will solve this in the manner of the second part the first section.

If we let Ω be zero, then the eigenstates of σ_z are eigenstates of the Hamiltonian. The field Hamiltonian (for constant ϵ) is given by

$$H_{\pm} = \frac{1}{2} \int (\pi - (\pm\epsilon(t)h(x)))^2 + (\partial_x\phi)^2 dx. \quad (50)$$

Defining $\tilde{\pi} = \pi - (\pm h(x))$, $\tilde{\pi}$ has the same commutation relations with π and ϕ as does π . Thus in terms of $\tilde{\pi}$ we just have the Hamiltonian for the free scalar field. The instantaneous minimum energy state is therefor the ground state energy for the free scalar field for both H_{\pm} . Thus the two states are degenerate in energy. In terms of the operators π and ϕ , these ground states are coherent states with respect to the vacuum state of the original uncoupled ($\epsilon = 0$) free field, with the displacement of each mode given by

$$a_k|\pm > = \pm i\epsilon(t)\frac{h(k)}{\sqrt{\omega_k}}|\pm > \quad (51)$$

or

$$|\pm > = \prod_k |\pm \alpha_k > |\pm >_{\sigma_3} \quad (52)$$

where the $|\alpha_k >$ are coherent states for the k^{th} modes with coherence parameter $\alpha_k = i\epsilon(t)\frac{h(k)}{\sqrt{\omega_k}}$, and the states $|\pm >_{\sigma_3}$ are the two eigenstates of σ_3 . (In the following I will eliminate the \prod_k symbol.) The energy to the next excited state in each case is just m , the mass of the free field.

We now introduce the $\Omega\sigma_x$ as a perturbation parameter. The two lowest states (and in fact the excited states) are two fold degenerate. Using degenerate perturbation theory to find

the new lowest energy eigenstates, we must calculate the overlap integral of the perturbation between the original degenerate states and must then diagonalise the resultant matrix to lowest order in Ω . The perturbation is $\frac{1}{2}\Omega\sigma_1$. All terms between the same states are zero, because of the $\langle \pm |_{\sigma_3} \sigma_1 | \pm \rangle_{\sigma_3} = 0$. Thus the only terms that survive for determining the lowest order correction to the lowest energy eigenvalues are

$$\frac{1}{2} \langle + | \Omega \sigma_1 | - \rangle = \frac{1}{2} \langle - | \Omega \sigma_1 | + \rangle^* \quad (53)$$

$$= \frac{1}{2} \Omega \prod_k \langle \alpha_k | - \alpha_k \rangle = \frac{1}{2} \Omega \prod_k e^{-2|\alpha_k|^2} \quad (54)$$

$$= \frac{1}{2} \Omega e^{-2 \int \epsilon(t)^2 |h(k)|^2 / \omega_k dk} = \frac{1}{2} \Omega J(t) \quad (55)$$

The eigenstates of energy thus have energy of $E(t)_{\pm} = E_0 \pm \frac{1}{2}\Omega J(t)$, and the eigenstates are $\sqrt{\frac{1}{2}}(|+ \rangle \pm |- \rangle)$. If epsilon varies slowly enough, the instantaneous energy eigenstates will be the actual adiabatic eigenstates at all times, and the time evolution of the system will just be in terms of these instantaneous energy eigenstates. Thus the system will evolve as

$$|\psi(t) \rangle = \sqrt{\frac{1}{2}} e^{-iE_0 t} \left((c_+ + c_-) e^{-i \int \frac{1}{2} \Omega J(t) dt} (|+ \rangle + |- \rangle) \right. \quad (56)$$

$$\left. + (c_- - c_+) e^{+i \int \frac{1}{2} \Omega J(t) dt} (|+ \rangle - |- \rangle) \right) \quad (57)$$

where the c_+ and c_- are the initial amplitudes for the $|+ \rangle_{\sigma_3}$ and $|- \rangle_{\sigma_3}$ states. The reduced density matrix for the spin system in the σ_3 basis can now be written as

$$\vec{\rho}(t) = (J(t)\rho_{01}(t), J(t)\rho_{02}(t), \rho_{03}(t)) \quad (58)$$

where $\vec{\rho}_0(t)$ is the density matrix that one would obtain for a free spin half particle moving under the Hamiltonian $J(t)\Omega\sigma_1$.

$$\begin{aligned} \rho_{01}(t) &= \rho_1(0) \\ \rho_{02}(t) &= \rho_2(0) \cos(\Omega \int J(t') dt') + \rho_3(0) \sin(\Omega \int J(t') dt') \\ \rho_{03}(t) &= \rho_3(0) \cos(\Omega \int J(t') dt') - \rho_2(0) \sin(\Omega \int J(t') dt') \end{aligned} \quad (59)$$

Thus if $J(t)$ is very small (i.e., ϵ large), we have a renormalized frequency for the spin system, and the off diagonal terms (in the σ_3 representation) of the density matrix are strongly suppressed by a factor of $J(t)$. Thus if we begin in an eigenstate of σ_3 the density matrix will begin with the vector $\vec{\rho}$ as a unit vector pointing in the 3 direction. As time

goes on the 3 component gradually decreases to zero, but the 2 component increases only to the small value of $J(t)$. The system looks almost like a completely incoherent state, with almost the maximal entropy that the spin system could have. However as we wait longer, the 3 component of the density vector reappears and grows back to its full unit value in the opposite direction, and the entropy drop to zero again. This cycle repeats itself endlessly with the entropy oscillating between its minimum and maximum value forever.

The decoherence of the density matrix (the small off diagonal terms) obviously represent a false loss of coherence. It represents a strong correlation between the system and the environment. However the environment is bound to the system, and essentially forms a part of the system itself, at least as long as the system moves slowly. However the reduced density matrix makes no distinction between whether or not the correlations between the system and the environment are in some sense bound to the system, or are correlations between the system and a freely propagating modes of the medium in which case the correlations can be extremely difficult to recover, and certainly cannot be recovered purely by manipulations of the system alone.

V. INSTANTANEOUS CHANGE

In the above I have assumed throughout that the system moves slowly with respect to the excitations of the heat bath. Let us now look at what happens in the spin system if we rapidly change the spin of the system. In particular I will assume that the system is as in section 1, a spin coupled only to the massive heat bath via the component σ_3 of the spin. Then at a time t_0 , I instantly rotate the spin through some angle θ about the 1 axis. In this case we will find that the environment cannot adjust rapidly enough, and at least a part of the loss of coherence becomes real, becomes unrecoverable purely through manipulations of the spin alone.

The Hamiltonian is

$$H = \frac{1}{2} \int \left((\pi(t, x) - \epsilon(t)h(x)\sigma_3)^2 + (\partial_x \phi(t, x))^2 + m^2 \phi(t, x) \right) dx + \theta/2 \delta(t - t_0) \sigma_1 \quad (60)$$

Until the time t_0 σ_3 is a constant of the motion, and similarly afterward. Before the time t_0 , the energy eigenstates state of the system are as in the last section given by

$$|\pm, t \rangle = \{ |+\rangle_{\sigma_3} |\alpha_k(t)\rangle \text{ or } |-\rangle_{\sigma_3} |-\alpha_k(t)\rangle \} \quad (61)$$

An arbitrary state for the spin–environment system is given by

$$|\psi\rangle = c_+|+\rangle + c_-|-\rangle \quad (62)$$

Now, at time t_0 , the rotation carries this to

$$\begin{aligned} |\phi(t_0)\rangle &= c_+(\cos(\theta/2)|+\rangle_{\sigma_3} + i\sin(\theta/2)|-\rangle_{\sigma_3})|\alpha_k(t)\rangle \\ &\quad + c_-(\cos(\theta/2)|-\rangle_{\sigma_3} + i\sin(\theta/2)|+\rangle_{\sigma_3})|-\alpha_k(t)\rangle \\ &= \cos(\theta/2)(c_+|+\rangle + c_-|-\rangle) \\ &\quad + i\sin(\theta/2)(c_+|-\rangle_{\sigma_3}|\alpha_k(t)\rangle - c_-|+\rangle_{\sigma_3}|-\alpha_k(t)\rangle) \end{aligned} \quad (63)$$

The first term is still a simple sum of eigenvectors of the Hamiltonian after the interaction. The second term, however, is not. We thus need to follow the evolution of the two states $|-\rangle_{\sigma_3}|\alpha_k(t_0)\rangle$ and $|+\rangle_{\sigma_3}|-\alpha_k(t_0)\rangle$. Since σ_3 is a constant of the motion after the interaction again, the evolution takes place completely in the field sector. Let us look at the first state first. (The evolution of the second can be derived easily from that for the first because of the symmetry of the problem.)

I will again work in the Heisenberg representation. The field obeys

$$\dot{\phi}_-(t, x) = \pi_-(t, x) + \epsilon(t)h(x) \quad (64)$$

$$\dot{\pi}_-(t, x) = \partial_x^2 \phi_-(t, x) - m^2 \phi_-(t, x) \quad (65)$$

with solution At the time t_0 the field is in the coherent state $|\alpha_k\rangle$. This can be represented by taking the field operator to be of the form

$$\phi_-(t_0, x) = \phi_0(t_0, x) \quad (66)$$

$$\pi_-(t_0, x) = \dot{\phi}_0(t_0, x) + \epsilon(t_0)h(x) \quad (67)$$

where the state $|\alpha_k\rangle$ is the vacuum state for the free field ϕ_0 . We can now solve the equations of motion for ϕ_- and obtain (again assuming that $\epsilon(t)$ is slowly varying)

$$\phi_-(t, x) = \phi_0(t, x) + 2\psi(t, x)\epsilon(t_0) \quad (68)$$

$$\pi_-(t, x) = \dot{\phi}_0(t, x) + 2\dot{\psi}(t, x)\epsilon(t_0) - \epsilon(t)h(x) \quad (69)$$

where $\psi(t_0, x) = 0$ and $\dot{\psi}(t_0, x) = h(x)$. Thus again, the field is in a coherent state set by both $2\epsilon(t_0)\psi$ and $\epsilon(t)h(x)$. The field ψ propagates away from the interaction region

determined by $h(x)$, and I will assume that I am interested in times t a long time after the time t_0 . At these times I will assume that $\int h(x)\psi(t, x)dx = 0$. (This overlap dies out as $1/\sqrt{mt}$. The calculations can be carried out for times nearer t_0 as well— the expressions are just messier and not particularly informative.)

Let me define the new coherent state as $|- \rangle_{\sigma_3} |\alpha_k(t) + \beta_k(t) \rangle$, where α_k is as before and

$$\beta_k(t) = 2\epsilon(t_0)\omega_k\tilde{\psi}(t, k) = 2i\epsilon(t_0)e^{i\omega_k t}\tilde{h}(k)/\omega_k \quad (70)$$

(The assumption regarding the overlap of $h(x)$ and $\psi(t)$ corresponds to the assumption that $\int \alpha_k^*(t)\beta_k(t)dk = 0$). Thus the state $|- \rangle_{\sigma_3} |\alpha_k \rangle$ evolves to the state $|- \rangle_{\sigma_3} |-\alpha_k + \beta_k(t) \rangle$. Similarly, the state $|+ \rangle_{\sigma_3} |-\alpha_k \rangle$ evolves to $|+ \rangle_{\sigma_3} |\alpha_k - \beta_k(t) \rangle$.

We now calculate the overlaps of the various states of interest.

$$\langle \alpha_k | \alpha_k \pm \beta_k \rangle = \langle -\alpha_k | -\alpha_k \pm \beta_k \rangle = e^{-\int |\beta_k|^2 dk} = J(t_0) \quad (71)$$

$$\langle -\alpha_k | \alpha_k \pm \beta_k \rangle = \langle \alpha_k | -\alpha_k \pm \beta_k \rangle = J(t)J(t_0) \quad (72)$$

$$\langle -\alpha_k + \beta_k | \alpha_k - \beta_k \rangle = \langle -\alpha_k - \beta_k | \alpha_k + \beta_k \rangle = J(t)J(t_0)^4 \quad (73)$$

The density matrix becomes

$$\rho_3 = \cos(\theta)\rho_{03} + \sin(\theta)J(t_0)\rho_{02} \quad (74)$$

$$\rho_1 = J(t) \left(\cos(\theta) + J^4(t_0) \sin(\theta) \right) \rho_{01} \quad (75)$$

$$\rho_2(t) = J(t) \left(-\sin(\theta)\rho_{03} + (\cos(\theta/2) - J^4(t_0) \sin(\theta))\rho_{02} \right) \quad (76)$$

where

$$\rho_{03} = \frac{1}{2}(|c_+|^2 - |c_-|^2) \quad (77)$$

$$\rho_{01} = \text{Re}(c_+c_-^*) \quad (78)$$

$$\rho_{02} = \text{Im}(c_+c_-^*) \quad (79)$$

If we now let $\epsilon(t)$ go slowly to zero again (to find the ‘real’ loss of coherence), we find that unless $\rho_{01} = \rho_{02} = 0$ the system has really lost coherence during the sudden transition. The maximum real loss of coherence occurs if the rotation is a spin flip ($\theta = \pi$) and ρ_{03} was zero. In that case the density vector dropped to $J(t_0)^4$ of its original value. If the density matrix was in an eigenstate of σ_3 on the other hand, the density matrix remained a coherent density matrix, but the environment was still excited by the spin.

We can use the models of a fast or a slow spin flip interaction to discuss the problem of the tunneling time. As Leggett et al argue[3], the spin system is a good model for the consideration of the behaviour of a particle in two wells, with a tunneling barrier between the two wells. One view of the transition from one well to the other is that the particle sits in one well for a long time. Then at some random time it suddenly jumps through the barrier to the other side. An alternative view would be to see the particle as if it were a fluid, with a narrow pipe connecting it to the other well- the fluid slowly sloshing between the two wells. The former is supported by the fact that if one periodically observes which of the two wells the particle is in, one sees it staying in one well for a long time, and then between two observations, suddenly finding it in the other well. This would, if one regarded it as a classical particle imply that the whole tunneling must have occurred between the two observations. It is as if the system were in an eigenstate and at some random time an interaction flipped the particle from one well to the other. However, this is not a good picture. The environment is continually observing the system. If it really moved rapidly from one to the other, the environment would see the rapid change, and would radiate. Instead, left on its own, the environment in this problem (with a mass much greater than the frequency of transition of the system) simply adjust continually to the changes in the system. The tunneling thus seems to take place continually and slowly.

VI. DISCUSSION

The high frequency modes of the environment lead to a loss of coherence (decay of the off-diagonal terms in the density matrix) of the system, but as long as the changes in the system are slow enough this decoherence is false— it does not prevent the quantum interference of the system. The reason is that the changes in the environment caused by these modes are essentially tied to the system, they are adiabatic changes to the environment which can easily be adiabatically reversed. Loosely one can say that coherence is lost by the transfer of information (coherence) from the system to the environment. However in order for this information to be truly lost, it must be carried away by the environment, separated from the system by some mechanism or another so that it cannot come back into the system. In the environment above, this occurs when the information travels off to infinity. Thus the loss of coherence as represented by the reduced density matrix is in some sense the maximum loss

of coherence of the system. Rapid changes to the system, or rapid decoupling of the system from the environment, will make this a true decoherence. However, gradual changes in the system or in the coupling to the external world can cause the environment to adiabatically track the system and restore the coherence apparently lost.

This is of special importance to understanding the effects of the environmental cutoff in many environments[3]. For “ohmic” or “superohmic” environments (where \hbar does not fall off for large arguments), one has to introduce a cutoff into the calculation for the reduced density matrix. This cutoff has always been a bit mysterious, especially as the loss of coherence depends sensitively on the value of this cutoff. If one imagines the environment to include say the electromagnetic field, what is the right value for this cutoff? Choosing the Plank scale seems silly, but what is proper value? The arguments of this paper suggest that in fact the cutoff is unnecessary except in renormalising the dynamics of the system. The behaviour of the environment at frequencies much higher than the inverse time scale of the system leads to a false loss of coherence, a loss of coherence which does not affect the actual coherence (ability to interfere with itself) of the system. Thus the true coherence is independent of cutoff.

As far as the system itself is concerned, one should regard it as “dressed” with a polarization of the high frequency components of the environment. One should regard not the system itself as important for the quantum coherence, but a combination of variables of the system plus the environment. What is difficult is the dependence of which the degrees of freedom of the environment are simply dressing and which are degrees of freedom which can lead to loss of coherence depends crucially on the motion and the interactions of the system itself. They are history dependent, not simply state dependent. This make it very difficult to simply find some transformation which will express the system plus environment in terms of variables which are genuinely independent, in the sense that if the new variable loose coherence, then that loss is real.

These observations emphasis the importance of not making too rapid conclusions from the decoherence of the system. This is especially true in cosmology, where high frequency modes of the cosmological system are used to decohere low frequency quantum modes of the universe. Those high frequency modes are likely to behave adiabatically with respect to the low frequency behaviour of the universe. Thus although they will lead to a reduced density matrix for the low frequency modes which is apparently incoherent, that incoherence

is likely to be a false loss of coherence.

Acknowledgments

I would like to thank the Canadian Institute for Advanced Research for their support of this research. This research was carried out under an NSERC grant 580441.

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- [1] A. O. Caldeira, A. J. Leggett *Physica* **121A** 587(1983), *Phys Rev* **A31** 1057 (1985) . See also the paper by W. Unruh, W. Zurek, *Phys Rev* **D40**1071(1989) where a field model for coherence instead of the oscillator model for calculating the density matrix of an oscillator coupled to a heat bath.
 - [2] Many of the points made here have also been made by A. Leggett. See for example A. J. Leggett in *Applications of Statistical and Field Theory Methods to Condensed Matter*(Proc. 1989 Nato Summer School, Evora, Portugal), ed D. Baeriswyl, A.R. Bishop, and J. Carmelo. Plenum Press (1990) and **Macroscopic Realism: What is it, and What do we know about it from Experiment** in *Quantum Measurement: Beyond Paradox* ed R. A. Healey, and G. Hellman, U. Minnesota Press (Minneapolis, 1998)
 - [3] See for example the detailed analysis of the density matrix of a spin 1/2 system in an oscillator heat bath, where the so called superohmic coupling to the heat bath leads to a rapid loss of coherence due to frequencies in the bath much higher than the frequency of the system under study. A.J. Leggett et al *Rev. Mod.Phys* **59** 1 (1987)
 - [4] This topic is a long standing one. For a review see R. Landauer and T. Martin, *Reviews of Modern Physics* **66** 217 (1994)